Direct detection of the relative strength of Rashba and Dresselhaus spin-orbit interaction: Utilizing the SU(2) symmetry

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We propose a simple method to detect the relative strength of Rashba and Dresselhaus spin-orbit interactions in quantum wells (QWs) without relying on the directional-dependent physical quantities. This method utilizes the two different critical gate voltages that leading to the remarkable signals of SU(2) symmetry, which happens to reflect the intrinsic-structure-inversion asymmetry of the QW. We support our proposal by the numerical calculation of in-plane relaxation times based on the self-consistent eight-band Kane model. We find that the two different critical gate voltages leading to the maximum spin-relaxation times [one effect of the SU(2) symmetry] can simply determine the ratio of the coefficients of Rashba and Dresselhaus terms. Our proposal can also be generalized to extract the relative strengths of the spin-orbit interactions in quantum-wire and quantum-dot structures.

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The spin-orbit interaction (SOI), which is a manifestation of the relativistic effect, transforms the electric fields into momentum-dependent effective magnetic fields, coupling the electron spin with electron orbital motion. The SOI provides us an efficient way to control the electron spin with electric fields instead of magnetic fields,[1,2] therefore plays an important role in realizing all-electrical controlled spintronic devices.[3] According to the different origins of SOI in semiconductor quantum structures, the SOI has been distinguished by the Rashba SOI (RSOI) arising from structure-inversion asymmetry (SIA) (Ref. 4) and the Dresselhaus SOI (DSOI) caused by bulk-inversion asymmetry (BIA), respectively.[5] These two types of SOI, yielding different effective SO magnetic field, leads to different behaviors of spin-transport properties and spin relaxation. Naturally, the ratio of Rashba and Dresselhaus coefficients (RD ratio) becomes a key parameter for understanding the spin-related phenomena and designing the future spintronic devices. Previously, the RD ratio can be determined by mapping the $\boldsymbol{k}$-dependent spin photocurrent,[6,7] in-plane spin-relaxation time,[8] the spin precession about the effective spin-orbit magnetic fields[9] into the components coming from DSOI and RSOI, or utilizing the anisotropic conductance of quantum wires in the presence of in-plane magnetic fields.[10] Therefore the above methods require exquisite measurement with respect of the crystallographic axis. Although these methods have been successfully used to study the relative strength of the Rashba and Dresselhaus SOIs in two-dimensional quantum well and heterostructures, the obtained RD ratio still holds certain ambiguity as pointed in Refs. 7, 8, and 10. Hence, finding a simple and accurate method to determine the RD ratio in all sorts of systems remains a challenging but important task.

In this paper, we propose a direct method that can separate the RSOI from DSOI and determine the RD ratio in an asymmetric [001]-oriented zinc blende quantum well (QW). Applying a gate voltage cross the QW to tune the total RSOI in this structure (see Fig. 1), we can find two different magnitudes of critical gate voltages to restore the exact SU(2) symmetry[11,12] by strengthening or canceling intrinsic RSOI existing in this QW. The difference between these two critical gate voltages extracts the exact information of intrinsic-structure-inversion asymmetry of this QW with bulk-inversion asymmetry separating apart. Therefore the two critical gate voltages can be used to determine the RD ratio in this asymmetric QW. Compare with the previous works, this proposal does not rely on measurements along specific directions[13] and is robust against all the effects that cannot change the structure-inversion asymmetry such as the isotropic impurity scattering. In addition, this proposal offers a general scheme that does not depend on a specific experimental technology and the dimensionality of the experimental sample, e.g., quantum wires and dots. A series of remarkable physical effects in quantum wells, wires, and dots[11,12,14–19] led by the SU(2) symmetry can be used to measure the critical gate voltages, consequently the RD ratio in these quantum structures.

First we give a picture of our proposal based on the single-band model of two-dimensional electron gas (2DEG) with two types of SOI, $H=(\hbar^2k^2)/(2m)+H_R+H_D$, where $H_R=\alpha(k_y\sigma_x-k_x\sigma_y)$ is the RSOI term, $H_D=\beta(k_y\sigma_y-k_x\sigma_x)+\gamma(k_y^2k_x\sigma_x-k_x^2k_y\sigma_y)$ is the DSOI term, and $\boldsymbol{k}=(k_x,k_y)$ is in-plane wave vector. Here, α is the linear Rashba coefficient, β and γ are the linear and cubic Dresselhaus coefficients, respectively. α can be tuned easily by changing the structure-inversion asymmetry, for instance, by gate voltage applied perpendicular to the QW plane.[1] While β and γ can be adjusted by changing the thickness of quantum wells.

The interplay between the RSOI and DSOI would lead to the anisotropy of optical and transport properties since the DSOI depends sensitively on the crystallographic orientations while the RSOI shows an isotropic behavior. If we tune gate voltage to satisfy $\alpha=\pm\beta$ (neglecting the cubic Dresselhaus term), the Hamiltonian of 2DEG show the exact SU(2) symmetry.[11] The exact SU(2) symmetry is a very unique property of quantum systems that the RSOI and DSOI happen to cancel each other for $\boldsymbol{k}$ along [110] or [1$\bar{1}$0] and is revealed to be robust against spin-independent disorder interactions.[11] As a consequence, the exact SU(2) symmetry would lead to a series of remarkable physical effects, such as a persistent spin helix exist in the sample,[11,12] a maximum spin lifetime for electron spins align along [1$\bar{1}$0] or [110]

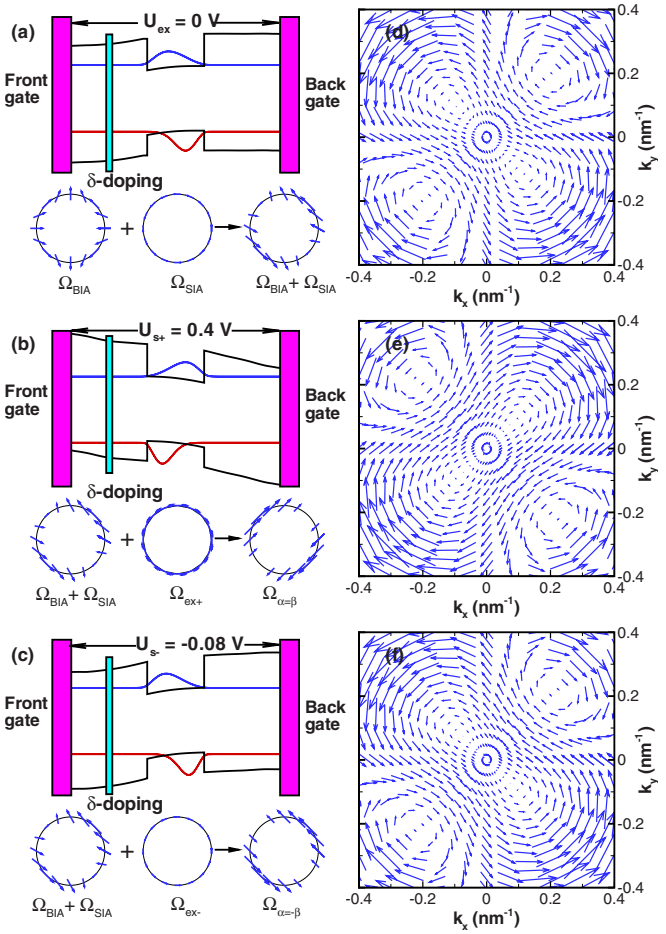

FIG. 1. (Color online) The calculated band profiles of an asymmetrically n-doped $Al_{0.3}Ga_{0.7}As/GaAs/Al_{0.3}Ga_{0.7}As$ QW and the electron (hole) probability distribution for different gate-voltage bias (a) $U_{ex}=0$ V, (b) $U_{s+}=0.4$ V, and (c) $U_{s-}=-0.08$ V. The sketches under each band profiles and the panels (d)–(f) shows the single-band schematic and self-consistent eight-band Kane model results, respectively. The doping concentration is fixed at $N_D=4.0\times10^{11}$ cm^{-2}.

direction,[14,15] the diminishing of the weak antilocalization[16] and the beating pattern of SdH oscillation.[17] Besides, in quasi-one-dimensional quantum wire and zero-dimensional quantum dot, the SU(2) symmetry could also induce strong physical effects, such as the conductance of a quantum wire shows strong anisotropy[18] and the maximum spin-relaxation time of a quantum dot.[19] All these physical effects can be used to determine the critical gate voltages that restore the SU(2) symmetry.

Notice that the SU(2) symmetry could be achieved by applying both positive and negative electric fields, each satisfying $\alpha=\pm\beta$. The total Rashba coefficient of QW structure in Fig. 1 can be viewed as a superposition of two parts $\alpha=\alpha_0+\alpha_{ex}$, where α_0 comes from the intrinsic SIA of the sample, e.g., the asymmetric doping or band profile and α_{ex} is introduced by the external electric field of the gate voltage. By sweeping the gate voltage, one can find two values $U_{s\pm}$ to meet the SU(2) symmetry conditions: $\alpha_0+\alpha_{s\pm}=\pm\beta$. Each $U_{s\pm}$ corresponds to $\alpha_{s\pm}$ for α_{ex}. If we neglect the difference in the dielectric constant between the well and barrier materials, we can simply assume $\alpha_{s\pm}$ to be proportional to $U_{s\pm}$. From the requirement restoring the SU(2) symmetry, one can find

$$\frac{\alpha_0}{\beta}=\frac{\alpha_{s+}+\alpha_{s-}}{\alpha_{s-}-\alpha_{s+}}=\frac{U_{s+}+U_{s-}}{U_{s-}-U_{s+}}. \quad (1)$$

Equation (1) demonstrate that, if there is no intrinsic SIA in the sample, i.e., $\alpha_0=0$, we should expect that $U_{s+}=-U_{s-}$; while if $\alpha_0\neq0$, we get $U_{s+}\neq-U_{s-}$. From a symmetry consideration, this conclusion is easy to understand because there would be no difference between $|U_{s\pm}|$ unless the [001] and [00$\bar{1}$] directions of the QW are asymmetric. This consideration guarantees our proposal to be robust against all the effects that does not change the symmetry of [001] and [00$\bar{1}$] directions, such as the isotropic impurity scattering. This conclusion can be also supported by an eight-band self-consistent calculation,[20] as shown in Fig. 1. In an asymmetrically doped $Al_{0.3}Ga_{0.7}As/GaAs/Al_{0.3}Ga_{0.7}As$ QW, the results show that the critical voltages to satisfy $\alpha=\pm\beta$ are 0.4 V and −0.08 V, respectively [see the panels (b) and (c)]. From the compositions of total effective spin-orbit magnetic field (the sketches under the band profile), one can see clearly that the difference between $|U_{s\pm}|$ comes from the intrinsic SIA. In Figs. 1(d)–1(f), we show the eight-band model results of effective spin-orbit magnetic field[20,21] which already takes the cubic Dresselhaus terms into account. Although the existence of cubic Dresselhaus terms might cause a different configuration rather than the exact SU(2) symmetry [see the panels (e) and (f)], the panel (e) still shows the mirror reflection symmetry with the panel (f), indicating that the total Rashba coefficient of these two panels are of the same magnitude (but with opposite signs). The intrinsic SIA still request two asymmetrical critical voltages to achieve the total ROI in panels (e) and (f). So the asymmetrical critical voltage always reflects the intrinsic SIA of QW, even taking account of the cubic Dresselhaus terms.

From Eq. (1) we can see that the RD ratio α_0/β can be obtained from $U_{s\pm}$ that leading to the SU(2) symmetry. Therefore, the experimental determination of α_0/β actually links with the of SU(2) symmetry. All previous works require the exact determination of the crystallographic orientation, e.g., the spin-relaxation time, spin splitting, the weak antilocalization effect, and Shubnikov-de Haas oscillation so that the effective magnetic fields induced by the RSOI and DSOI can cancel each other.[14–19] In our proposal, one can tune the gate voltage to find the two critical values $U_{s\pm}$, which can be detected experimentally from time-resolved photoluminescence spectrum in which one extracts the spin lifetime. Notice that our proposal is not limited to the dimensionality and can also be applied to quasi-one-dimensional quantum wires and zero-dimensional quantum dot.

Next, we will take the in-plane D'yakonov-Perel' (DP) spin-relaxation times (SRT)[22] as an example to demonstrate the validity of our proposal. This calculation is based on a self-consistent eight-band Kane model.[20,21] The band parameters can be found in Ref. 23 and the BIA Kane parameter B_0 are obtained from 14-band effective mass model in Ref. 24. In Figs. 2(a) and 2(c) we exhibit the calculated SRT for

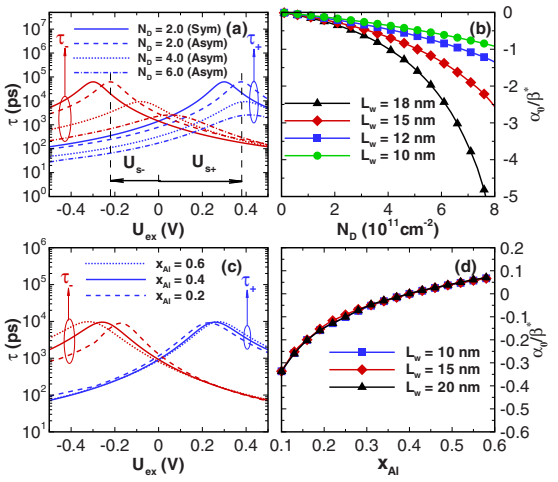

FIG. 2. (Color online) (a) Calculated in-plane SRTs $\tau_{\pm}$ as a function of gate voltage in n-doped $Al_{0.3}Ga_{0.7}As/GaAs/Al_{0.3}Ga_{0.7}As$ QW with different doping conditions: symmetrical doping (solid lines) and asymmetrical doping with different concentrations (unit: 10^{11} cm^{-2} dashed, dotted, and dashed-dotted lines), respectively. (c) $\tau_{\pm}$ as a function of gate voltage in $Al_xGa_{1-x}As/GaAs/Al_{0.3}Ga_{0.7}As$ QW with different barrier Al composition x. [(b) and (d)] α_0/β^* as a function of the doping concentration and Al composition, respectively, for different thicknesses of QWs.

electron spin along [110] or [1$\bar{1}$0] (denoted by τ_+ and τ_-, respectively) in 15 nm n-doped GaAs/AlGaAs QWs with different doping conditions and Al compositions of barrier (i.e., asymmetrical barrier heights), respectively. In order to understand the numerical results, we introduce the analytical results of DP SRTs at $T=0$ K (Ref. 14)

$$\frac{1}{\tau_{\pm}} = \frac{2\tau_1}{\hbar^2}\left[(\alpha \mp \beta)^2 k_F^2 - \frac{1}{2}\gamma(\beta \mp \alpha)k_F^4 + \frac{1+\tau_3/\tau_1}{16}\gamma^2 k_F^6\right], \quad (2)$$

where k_F is the Fermi wave vector of electron and a typical momentum scattering time $\tau_1=\tau_3=0.1$ ps is taken. By sweeping the gate voltage, the total Rashba coefficient α change linearly. So one can find the maximum SRT $\tau_{\pm}^{\max}=8\hbar^2/(k_F^6\gamma^2\tau_3)$ when $\alpha=\pm(\beta-\frac{1}{4}k_F^2\gamma)$, corresponding to the peaks of $\tau_{\pm}$ in Fig. 2(a). The two peaks are symmetric with respect to the zero voltage ($|U_{s+}|=|U_{s-}|$) for a symmetrical QW (see the solid lines) but asymmetric ($|U_{s+}|\neq|U_{s-}|$) (see the dashed, dotted, and dashed-dotted lines) for asymmetrical QWs. Different from the k-linear SOI model which neglects the cubic Dresselhaus term, $\tau_{\pm}$ would not go infinite and the maximum values do not show at $\alpha=\pm\beta$. Similar to the analysis on the spin-galvanic effect by Ganichev *et al.*,[6] we can take the $\beta^*=\beta-\frac{1}{4}k_F^2\gamma$ as the renormalized Dresselhaus coefficient for k-linear SOI model. Therefore we actually get α_0/β^* from Eq. (1) rather than α_0/β and the difference between β and β^* comes from the contribution of cubic Dresselhaus term. In Figs. 2(b) and 2(d) we display α_0/β^* determined from the critical gate voltage that lead to the maximum in-plane SRT. As expected, α_0/β^* increase with increasing the asymmetrical doping concentration or the composition difference between the left and right barriers. For the QWs with different doping concentrations, α_0/β^* also increase with increasing the well width, since $\beta^*\sim\beta\approx\gamma(\frac{\pi}{w})^2$. For the QWs with different compositions of barriers, α_0/β^* turns to be not sensitive to the change of well width, because α_0 are very small in these cases.

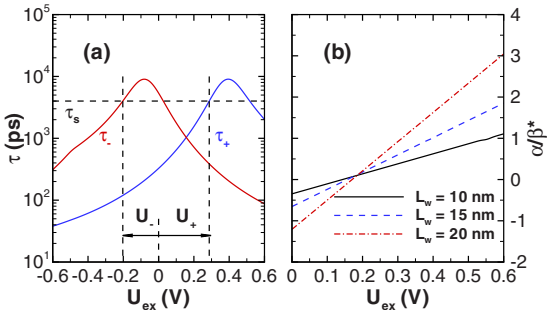

FIG. 3. (Color online) (a) The in-plane SRT $\tau_{\pm}$ as a function of gate voltage in 15-nm asymmetrically n-doped $Al_{0.3}Ga_{0.7}As/GaAs/Al_{0.3}Ga_{0.7}As$ QW ($N_D=4\times10^{11}$ cm^{-2}) and (b) α/β^* determined by Eq. (3) as a function of gate voltage for QWs with different well widths.

In addition to the intrinsic Rashba coefficient α_0, it is also possible to determine the relative strength of total Rashba coefficient α and Dresselhaus coefficient β in the QW through the in-plane SRTs. From Eq. (2) we can find the condition for the in-plane SRTs $\tau_{\pm}$ to reach a same value τ_s satisfying $\alpha_0+\alpha_{\pm}=\pm\alpha$. As shown in Fig. 3(a), for a $\tau_s<\tau_{\pm}^{\max}$, we can find the gate voltages $U_{\pm}$ corresponding to $\tau_{\pm}=\tau_s$. Usually, there are two different gate voltages for each τ_+ and τ_-. So we must limit that if $|U_+|>|U_{s+}|$ ($|U_+|<|U_{s+}|$), we choose $|U_-|>|U_{s-}|$ ($|U_-|<|U_{s-}|$). Combine these conditions and Eq. (1), we can get

$$\frac{\alpha}{\beta^*} = \frac{(U_{s+}+U_{s-})(U_- - U_+)}{(U_{s-}-U_{s+})(U_+ + U_-)}. \quad (3)$$

If the QW is inversion symmetric, we should have

$$\frac{\alpha}{\beta^*} = \frac{U_- - U_+}{U_{s+}-U_{s-}} = \frac{U_{\pm}}{U_{s\pm}}. \quad (4)$$

In Fig. 3(b) we plot α/β^* as a function of gate voltage in asymmetrically n-doped GaAs/AlGaAs QWs with different well widths. The figure shows α/β^* increase almost linearly with gate voltage, which is consistent with the analytical results of Rashba coefficient in the previous work.[25] The slope of α/β^* for narrow wells are smaller than that of thick wells because β^* is larger in narrow wells.

In Fig. 4(a), we investigate the temperature effect on the in-plane SRT. As we shown in the figure, the in-plane SRT peaks which characters the emergence of SU(2) symmetry would be gradually smeared out when temperature increases from $T=4-150$ K or even higher. That is because of the blurring of the Fermi surface with increasing temperature.[21] We may also associate that many of other SU(2) symmetry phenomenons could disappear at high temperature due to the blurring of the Fermi surface. So it is suggested that the SU(2) symmetry phenomenons should be observed at $T<77$ K. In Fig. 4(b) we compare the RD ratios α_0/β^* obtained by Eq. (1) and that obtained by fitting parameters directly from the spin splitting. We find that the ratios α_0/β^*

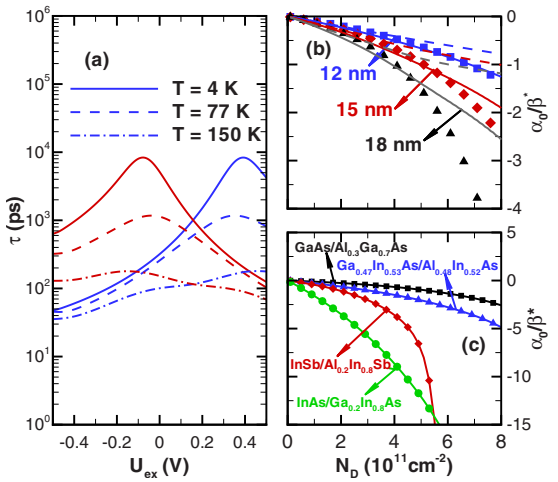

FIG. 4. (Color online) (a) Calculated in-plane SRT $\tau_{\pm}$ as a function of gate voltage in an 15-nm asymmetrically doped ($N_D=4\times10^{11}$ cm^{-2}) $Al_{0.3}Ga_{0.7}As/GaAs/Al_{0.3}Ga_{0.7}As$ QW at different temperatures. (b) α_0/β^* as a function of asymmetric doping concentration in QW with different well widths $L_w=12,15,18$ nm. The triangle, diamond, and square dots are obtained from Eq. (1). The solid and dashed lines are α_0/β^* and α_0/β obtained by fitting parameters directly from the eight-band spin splitting, respectively. (c) The RD Ratio α_0/β^* obtained from Eq. (1) as a function of asymmetrical-doping concentration N_D for different material of QWs.

agree well with the results fitted from the spin splitting at low-doping concentration and are especially good for narrow QWs. For heavily doped QW with thick wells, α_0/β^* obtained by the SRT calculation could deviate from that the fitting results of the spin splitting. The reason is when the doping concentration is high, the charge distribution under finite gate voltage would cause the change of the internal electric field and hence the value of α_0. However, as we see in the Fig. 4(b), the charge redistribution effect is very limited at light doping ($N_D<4\times10^{11}$ cm^{-2}) and narrow QWs ($L_w<15$ nm) [see Fig. 4(b)], so that α_0/β^* obtained by the critical gate-voltages measurement are more accurate in these cases. In Fig. 4(b) we also show the difference between α_0/β and α_0/β^*, which comes from the contribution of the cubic Dresselhaus terms and is very small (<0.2) when $N_D<3\times10^{11}$ cm^{-2}, demonstrating that the single band model with k-linear SOI coefficients are valid at low doping concentration. In Fig. 4(c) we show the calculated RD ratio α_0/β^* for different QWs. Though these BIA Kane parameters B_0 are still in a big uncertainty today, we can still see that for the narrow band-gap QW, such as InAs/GaInAs and InSb/AlInSb, the relative strength of RSOI are much larger than that of middle band-gap QWs GaAs/AlGaAs and GaInAs/AlInAs. This is because the RSOI comes from the interband coupling of conduction and valence bands, which is much stronger in narrow band-gap materials.

In summary, we proposed a simple and direct method to separate the intrinsic RSOI from DSOI. The relative strength of RSOI and DSOI can be determined by the critical gate voltages that restores the SU(2) symmetry in 2DEG. The SU(2) symmetry leads to a series of characteristic physical effects, such as the maximum in-plane spin-relaxation time, the persistent spin helix, and so on. Through the in-plane spin-relaxation time calculation based on the self-consistent eight-band model, we demonstrate our proposal is valid and can be used to detect the strengths of the SOIs in quantum wells, wires, and dots utilizing the SU(2) symmetry. Our proposal offers a general scheme that many experimental techniques could be used to determine this important parameter and facilitate us to manually control the spin degree of freedom.

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